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The Optimization of the Generalized Coplanar Impulsive Maneuvers (Two Impulses, Three Impulses and One Tangent Burn)

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Abstract: **The orbit transfer problems using impulsive thrusters have attracted researchers for a long time [3]. One of the objectives in these problems is to find the optimal fuel orbit transfer between two orbits, generally inclined eccentric orbits. The optimal two-impulse orbit transfer problem poses multiple local optima, and classical optimization methods find only local optimum solution. McCue [7] solved the problem of optimal two-impulse orbit transfer using a combination between numerical search and steepest descent optimization procedures. The transfer of satellites in too high orbits as geosynchronous one (geostationary), usually is achieved firstly by launching the satellite in Low Earth Orbit (LEO) (Parking orbit), then in elliptical transfer orbit and finally to the final orbit (Working orbit). The three steps process is known as Hohmann transfer. The Hohmann transfer which involves two circular orbits with different orbital inclinations is known as non-coplanar Hohmann transfer. If both orbital planes are aligned the Hohmann transfer is known as coplanar what is further considered in this paper. In terms of propellant consumptions the Hohmann transfer is the best known transfer to be applied when transferring between elliptical coplanar orbits. For transfer between elliptical coplanar orbits, the given information usually consists of the altitude of perigee and apogee of the initial and the altitude of perigee and apogee of the final orbits. The velocity to be applied into two orbit points in order to attain the dedicated final orbit is analyzed.**

The aim of this paper is compare between three types of coplanar impulsive transfer (two impulses, three impulses and one tangent burn) and conclude about the velocity changes for these types under relation between initial low Earth altitudes and final orbit. For the relation between initial orbit altitudes and final orbit altitude, the velocities to be applied in process of Hohmann transfer are simulated. From respective simulations, the velocity variations on dependence of this relation are derived. And the time of flight is considered too. The problem of spacecraft orbit transfer with minimum fuel consumption is considered, in terms of testing numerical solutions.

Keywords: **LEO; Satellite; Orbit; Hohmann Transfer; One Tangent Burn; Coplanar Maneuver; Impulsive Maneuver.**

1. INTRODUCTION

R. H. Goddard (1919) was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption.

After that, there is the very important work done by Hohmann (1925). He solved the problem of minimum ΔV transfers between two circular coplanar orbits. His results are largely used nowadays as a first approximation of more complex models and it was considered the final solution of this problem until 1959.

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The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal (1965) [6]. Smith (1959) showed results for some other special cases, like coaxial and quasi-coaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits.

Hohmann type transfers between non-coplanar orbits are discussed in several papers, like McCue (1962) [7] that studied a transfer between two elliptic inclined orbits including the possibility of rendezvous; or Eckel and Vinh (1984) [2] that solved the same problem with time or fuel fixed. When launching and then consolidating the satellite on its own high circular orbit (ex. geosynchronous), the needed satellite's propellant mass must be minimized. The Hohmann transfer is well known for the minimum of propellant mass used for satellite transfer into high orbits.

The Hohmann transfer is the best transfer to be used when transferring between circular coplanar orbits [3]. For transfer between circular coplanar orbits, the information usually given consists of the radii of the initial and final orbits. Due to the reversibility of orbits, Hohmann transfer orbits also work to bring a spacecraft from the higher orbit into the lower one. The Hohmann transfer orbit is based on two instantaneous velocity changes. The transfer consists of a velocity impulse on an initial circular orbit, in the direction of motion and collinear with velocity vector, which propels the space vehicle into an elliptical transfer orbit. The second velocity impulse also in the direction of motion is applied at apogee of the transfer orbit which propels the space vehicle into a final circular orbit at the final altitude [3, 7].

In this paper we will compare between three types of coplanar impulsive transfer (two impulses, three impulses and one tangent burn) and conclude about the velocity changes for these types under relation between initial low Earth altitudes and final orbit. For the relation between initial orbit altitudes and final orbit altitude, the velocities to be applied in process of Hohmann transfer are simulated. From respective simulations, the velocity variations on dependence of this relation are derived. And the time of flight is considered too. The problem of spacecraft orbit transfer with minimum fuel consumption is considered, in terms of testing numerical solutions.

Under the first section the elliptic orbit is generally considered. Further, through implementation of the single normalized radius, both velocity impulses to be applied at perigee and apogee of the Hohmann transfer and bi-elliptic Hohmann transfer orbit are analyzed then using one tangent burn to make transfer between two elliptic orbits. Considering different initial low Earth orbit altitudes respective velocities are calculated. Finally, these results are discussed and closed by conclusions.

2. DEFINITION OF THE PROBLEM

The problem to be studied is that of optimal transfer of a rocket vehicle between a pair of coplanar elliptical orbits, employing two or three impulsive thrusts or one tangent burn. The objective of this problem is to modify the orbit of a given spacecraft. The problem is to find how to choose the optimum method (two impulses, three impulses and one tangent burn) to transfer the spacecraft between two given orbits in a way that the velocity required is minimum and fuel consumed is minimum with the time of flight is restricted and there is on restriction on the spacecraft which can leave and arrive at any point in the given initial and final orbits.

3. ELLIPTIC ORBIT

The path of the satellite's motion is an orbit. Generally, the orbits of communication satellites are ellipses laid on the orbital plane defined by space orbital parameters. These parameters (Kepler elements) determine the position of the orbital plane in space, the location of the orbit within orbital plane and finally the position of the satellite in the appropriate orbit [1, 8].

The elliptic orbit is determined by the semi-major axis which defines the size of an orbit, and the eccentricity which defines the orbit's shape. Orbits with no eccentricity are known as circular orbits. The elliptic orbit shaped as an ellipse, with a maximum extension from the Earth center at the apogee (r_a) and the minimum at the perigee (r_p) is presented in Figure (1).

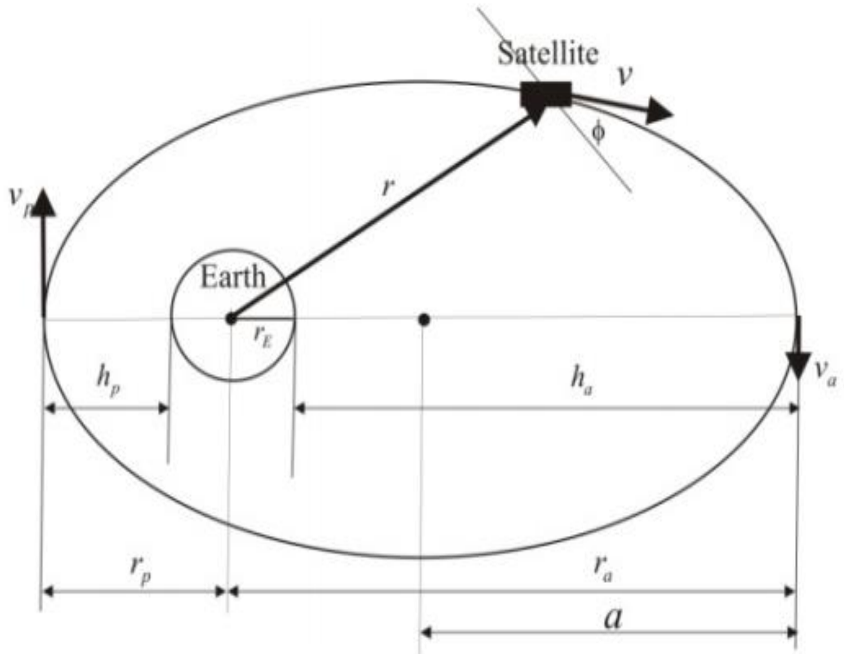

Figure (1): Major parameters of an elliptic orbit.

The orbit's eccentricity is defined as the ratio of difference to sum of apogee (ra) and perigee (rp) radii as, [1] - [8].

$$e = \frac{r_a - r_p}{r_a + r_p} \quad (1)$$

Applying geometrical features of ellipse yield out the relations between semi major axis, apogee and perigee:

$$r_p = a(1 - e) \quad (2)$$

$$r_a = a(1 + e) \quad (3)$$

$$2a = r_a + r_p \quad (4)$$

Both, r_p and r_a are considered from the Earth's center. Earth's radius is

$r_E = 6378$ km . Then, the altitudes (highs) of perigee and apogee are:

$$H_p = r_p - r_E \quad (5)$$

$$H_a = r_a - r_E \quad (6)$$

Different methods are applied for satellite injection missions. Goal of these methods is to manage and control the satellite to safely reach the low Earth orbit, and then to the transfer elliptical orbit and finally the final orbit [4] - [5]. The Hohmann transfer is considered as the most convenient. The specific orbit implementation depends on satellite's injection velocity. The orbit implementation process on the best way is described in terms of the cosmic velocities. Based on Kepler's laws, considering an elliptic orbit, the satellite's velocity at the perigee and apogee point.

4. COPLANAR IMPULSIVE MANEUVERS

1) Two impulses tangent (Hohmann transfer)

To transfer from elliptical orbit (initial) to elliptical orbit (final) by Hohmann transfer (two tangent impulses), we have two cases:

a) The first pulse in perigee point of initial orbit and the second pulse in apogee point of final orbit, figure (2).

b) The first pulse in apogee point of initial orbit and the second pulse in perigee point of final orbit, figure (3).

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➢ ***Cases (a)***

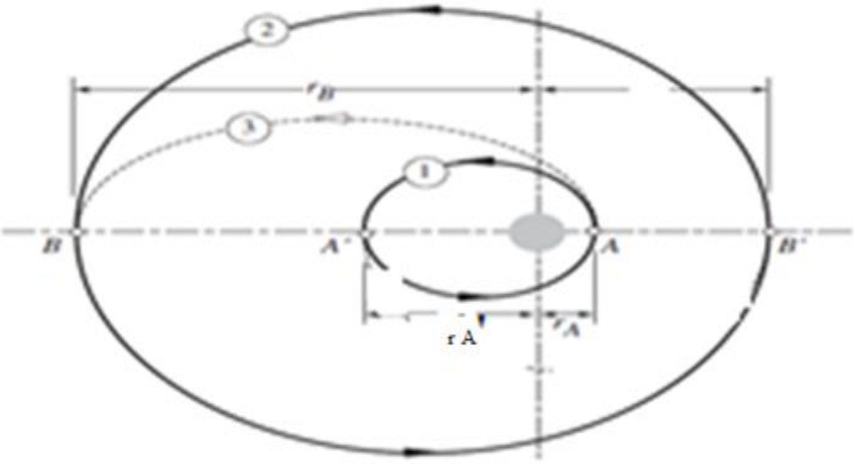

Figure (2): Hohmann transfer from perigee point of initial orbit

According to the figure (2) we have

- ***Delta-V for transfer***

  - At point A

$$v_A)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{p1}}\,(km/sec)\quad \text{For orbit (1)} \qquad (7)$$

$$v_A)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{p1}}\,(km/sec)\quad \text{For orbit (3)} \qquad (8)$$

$$\therefore \Delta V_A = |v_A)_3 - v_A)_1|\,(km/sec) \qquad (9)$$

  - At point B

$$v_B)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p2}}{r_{a2}+r_{p2}}}}{r_{a2}}\,(km/sec)\quad \text{For orbit (2)} \qquad (10)$$

$$v_B)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{a3}}\,(km/sec)\quad \text{For orbit (3)} \qquad (11)$$

$$\therefore \Delta V_B = |v_B)_3 - v_B)_2|\,(km/sec) \qquad (12)$$

The total delta-V requirement for this Hohmann transfer is

$$\Delta V_{total} = |\Delta V_A| + |\Delta V_B|\,(km/sec) \qquad (13)$$

- ***The time of flight***

The semi-major axis of the transfer ellipse is

$$a_t = \frac{1}{2}\left(r_{a2} + r_{p1}\right)\,(km) \qquad (14)$$

$$T = \frac{2\pi}{\sqrt{\mu}}\,a_t^{\frac{3}{2}}\,(sec) \qquad (15)$$

The time of flight for this hohmann transfer is

$$\therefore \Delta t = \frac{1}{2}T\,(sec) \qquad (16)$$

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- ***Cases (b)***

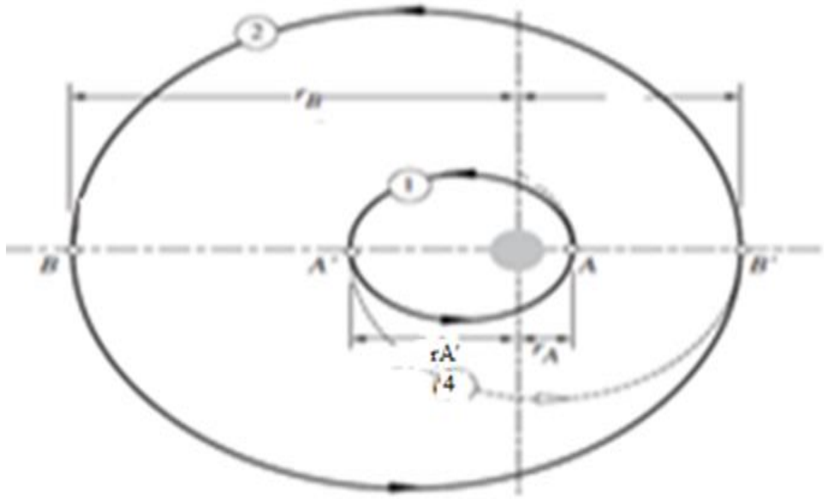

Figure (3): Hohmann transfer from apogee point of initial orbit

According to the figure (3) we have

- ***Delta-V for transfer***

○ At point A·

$$v_{A^\cdot})_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{a1}} \text{ (km/sec)} \quad \text{For orbit (1)} \qquad (17)$$

$$v_{A^\cdot})_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p2}}{r_{a1}+r_{p2}}}}{r_{a1}} \text{ (km/sec)} \quad \text{For orbit (4)} \qquad (18)$$

$$\therefore \Delta V_{A^\cdot} = |v_{A^\cdot})_1 - v_{A^\cdot})_4| \text{ (km/sec)} \qquad (19)$$

○ At point B·

$$v_{B^\cdot})_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p2}}{r_{a2}+r_{p2}}}}{r_{p2}} \text{ (km/sec)} \quad \text{For orbit (2)} \qquad (20)$$

$$v_{B^\cdot})_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p2}}{r_{a1}+r_{p2}}}}{r_{p2}} \text{ (km/sec)} \quad \text{For orbit (4)} \qquad (21)$$

$$\therefore \Delta V_{B^\cdot} = |v_{B^\cdot})_4 - v_{B^\cdot})_2| \text{ (km/sec)} \qquad (22)$$

The total delta-V requirement for this Hohmann transfer is

$$\Delta V_{total} = |\Delta V_{A^\cdot}| + |\Delta V_{B^\cdot}| \text{ (km/sec)} \qquad (23)$$

- ***The time of flight***

The semi-major axis of the transfer ellipse is

$$a_t = \frac{1}{2}\left(r_{p2} + r_{a1}\right) \text{ (km)} \qquad (24)$$

$$T = \frac{2\pi}{\sqrt{\mu}} a_t^{\frac{3}{2}} \text{ (sec)} \qquad (25)$$

The time of flight for this hohmann transfer is

$$\therefore \Delta t = \frac{1}{2} T \text{ (sec)} \qquad (26)$$

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2) Three impulses tangent (Bi-Elliptic Hohmann transfer)

To transfer from elliptical orbit (initial) to elliptical orbit (final) by Bi-Elliptic Hohmann transfer (three tangent impulses), we have two cases:

a) The first pulse in perigee point of initial orbit (point A) and the second pulse in apogee point of transfer orbit (point C) and third pulse in perigee point of final orbit (point B), Figure (4).

b) The first pulse in apogee point of initial orbit (point A) and the second pulse in apogee point of transfer orbit (point C) and third pulse in apogee point of final orbit (point B), Figure (5).

- ***Cases (a)***

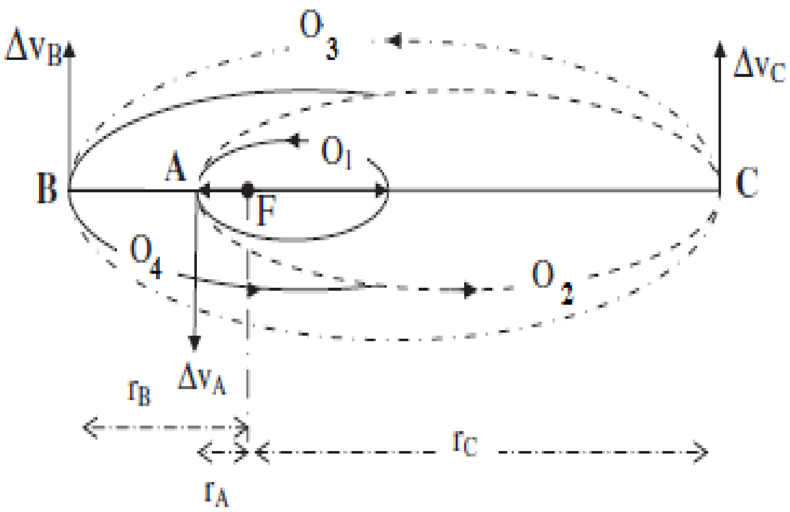

Figure (4): Bi-Elliptic Hohmann transfer from perigee point of initial orbit

According to the figure (4) we have

- ***Delta-V for transfer***

○ At point A

$$v_A)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{p1}} \text{(km/sec)} \quad \text{For orbit (1)} \qquad (27)$$

$$v_A)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{p1}} \text{(km/sec)} \quad \text{For orbit (2)} \qquad (28)$$

$$\therefore \Delta V_A = |v_A)_2 - v_A)_1| \text{ (km/sec)} \qquad (29)$$

○ At point C

$$v_C)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p1}}{r_{a2}+r_{p1}}}}{r_{a2}} \text{(km/sec)} \quad \text{For orbit (2)} \qquad (30)$$

$$v_C)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p4}}{r_{a2}+r_{p4}}}}{r_{a2}} \text{(km/sec)} \quad \text{For orbit (3)} \qquad (31)$$

$$\therefore \Delta V_C = |v_C)_3 - v_C)_2| \text{ (km/sec)} \qquad (32)$$

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- At point B

$$v_B)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{p4}}{r_{a2}+r_{p4}}}}{r_{p4}} \text{ (km/sec)} \quad \text{For orbit (3)} \qquad (33)$$

$$v_B)_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a4}r_{p4}}{r_{a4}+r_{p4}}}}{r_{p4}} \text{ (km/sec)} \quad \text{For orbit (4)} \qquad (34)$$

$$\therefore \Delta V_B = |v_B)_3 - v_B)_4| \text{ (km/sec)} \qquad (35)$$

The total delta-V requirement for this Bi-elliptic Hohmann transfer is

$$\Delta V_{total} = |\Delta V_A| + |\Delta V_C| + |\Delta V_B| \text{ (km/sec)} \qquad (36)$$

- ***The time of flight***

The semi-major axis of the transfer ellipse is

$$a_{t1} = \frac{1}{2}(r_{p1} + r_{a2}) \text{ (km)} \qquad (37)$$

$$a_{t2} = \frac{1}{2}(r_{p4} + r_{a2}) \text{ (km)} \qquad (38)$$

$$T = \frac{2\pi}{\sqrt{\mu}} a_{t1}^{\frac{3}{2}} + \frac{2\pi}{\sqrt{\mu}} a_{t2}^{\frac{3}{2}} \text{ (sec)} \qquad (39)$$

The time of flight for this Bi-elliptic Hohmann transfer is

$$\therefore \Delta t_{bi-elliptical} = \frac{1}{2} T \text{ (sec)} \qquad (40)$$

- ***Cases (b)***

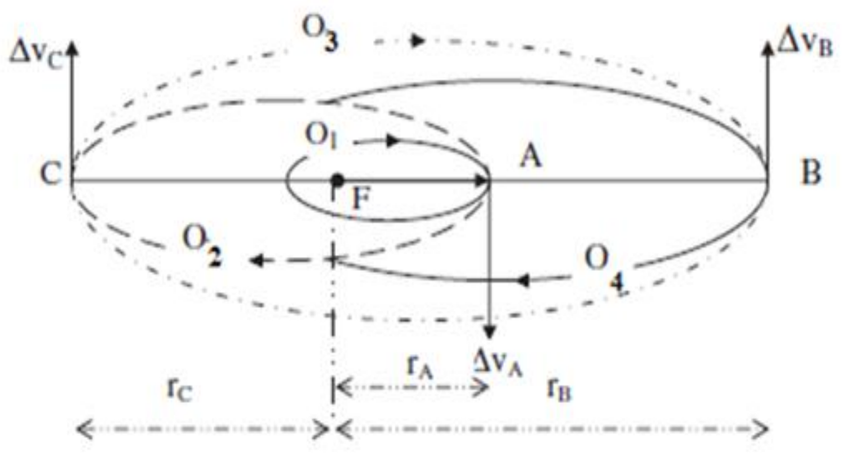

Figure (5): Bi-Elliptic Hohmann transfer from perigee point of initial orbit

According to the figure (5) we have

- ***Delta-V for transfer***

- At point A

$$v_A)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{a1}} \text{ (km/sec)} \quad \text{For orbit (1)} \qquad (41)$$

$v_A)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a1}}{r_{a2}+r_{a1}}}}{r_{a1}}$ (km/sec) For orbit (2) (42)

$\therefore \Delta V_A = |v_A)_2 - v_A)_1|$ (km/sec) (43)

- At point C

$v_C)_2 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a1}}{r_{a2}+r_{a1}}}}{r_{a2}}$ (km/sec) For orbit (2) (44)

$v_C)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a4}}{r_{a2}+r_{a4}}}}{r_{a2}}$ (km/sec) For orbit (3) (45)

$\therefore \Delta V_C = |v_C)_3 - v_C)_2|$ (km/sec) (46)

- At point B

$v_B)_3 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a2}r_{a4}}{r_{a2}+r_{a4}}}}{r_{a4}}$ (km/sec) For orbit (3) (47)

$v_B)_4 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a4}r_{p4}}{r_{a4}+r_{p4}}}}{r_{a4}}$ (km/sec) For orbit (4) (48)

$\therefore \Delta V_B = |v_B)_3 - v_B)_4|$ (km/sec) (49)

The total delta-V requirement for this Bi-elliptic Hohmann transfer is

$\Delta V_{total} = |\Delta V_A| + |\Delta V_C| + |\Delta V_B|$ (km/sec) (50)

- ***The time of flight***

The semi-major axis of the transfer ellipse is

$a_{t1} = \frac{1}{2}(r_{a1} + r_{a2})$ (km) (51)

$a_{t2} = \frac{1}{2}(r_{a2} + r_{a4})$ (km) (52)

$T = \frac{2\pi}{\sqrt{\mu}} a_{t1}^{\frac{3}{2}} + \frac{2\pi}{\sqrt{\mu}} a_{t2}^{\frac{3}{2}}$ (sec) (53)

The time of flight for this Bi-elliptic Hohmann transfer is

$\therefore \Delta t_{bi-elliptical} = \frac{1}{2} T$ (sec) (54)

3) One tangent burn

As the name implies, a one-tangent burn has one tangential burn and one non-tangential burn. This method reduces the transfer time of the Hohmann techniques but increases ΔV

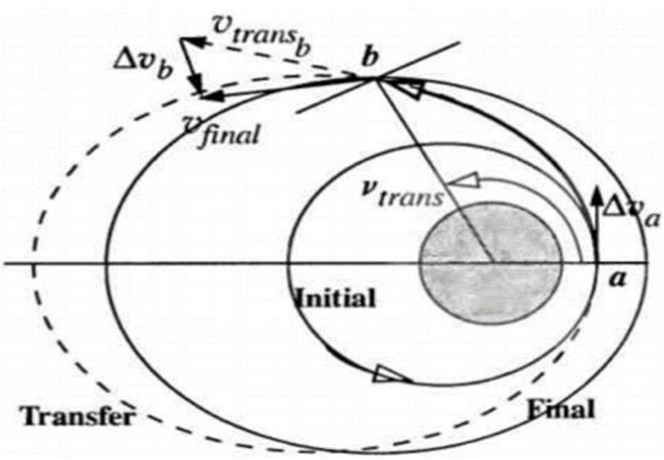

Figure (6): One tangent burn transfer

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According to the figure (6) we have

$$R = \frac{r_{initial}}{r_{final}} \quad (55)$$

$$R = \frac{r_{p1}}{r_{p2}} \quad (56)$$

Eccentricity and semi major axis of transfer orbit are

$$\therefore e_{trans} = \frac{R-1}{\cos \upsilon_{trans} - R} \quad (57)$$

$$\text{and } a_{trans} = \frac{r_{p1}}{1-e_{trans}} \quad (58)$$

- ***Delta-V for transfer***

  - At point a

$$V_a)_1 = \frac{\sqrt{(2\mu)}\sqrt{\frac{r_{a1}r_{p1}}{r_{a1}+r_{p1}}}}{r_{p1}} \text{(km/sec)} \quad \text{For orbit (1)} \quad (59)$$

$$V_{trans\ a})_3 = \sqrt{\frac{(2\mu)}{r_{p1}} - \frac{\mu}{a_{trans}}} \text{(km/sec)} \quad \text{For orbit (3)} \quad (60)$$

$$\therefore \Delta V_a = |V_{trans\ a})_3 - V_{Aa})_1| \text{ (km/sec)} \quad (61)$$

  - At point b

$$a_2 = \frac{r_{a2}+r_{p2}}{2} \quad (62)$$

$$r_b = \frac{a_{trans}(1-e_{trans}^2)}{1+e_{trans}\cos(v_{trans})} \quad (63)$$

$$V_b)_2 = \sqrt{\left(\frac{(2\mu)}{r_b} - \frac{\mu}{a_2}\right)} \text{ (km/sec)} \quad \text{For orbit (2)} \quad (64)$$

$$V_{trans\ b})_3 = \sqrt{\left(\frac{(2\mu)}{r_b} - \frac{\mu}{a_{trans}}\right)} \text{(km/sec)} \quad \text{For orbit (3)} \quad (65)$$

$$\tan \varphi_{trans\ b} = \frac{e_{trans} * \sin(v_{trans\ b})}{1+(e_{trans} * \cos(v_{trans\ b}))} \quad (66)$$

$$\Delta V_b = \sqrt{[V_b)_2]^2 + [V_{trans\ b})_3]^2 - [2 * V_b)_2 * V_{trans\ b})_3 * \cos(\varphi_{trans\ b})]} \quad (67)$$

The total delta-V requirement for this transfer is

$$\Delta V_{total} = |\Delta V_a| + |\Delta V_b| \text{(km/sec)} \quad (68)$$

- ***The time of flight***

$$\cos(E) = \frac{e_{trans} + \cos(v_{trans\ b})}{1+(e_{trans} * \cos(v_{trans\ b}))} \quad (69)$$

$$t_{trans} = \sqrt{\frac{a_{trans}^3}{\mu}} \{(2k\pi) + (E - (e_{trans} * \sin(E))) - (E_0 - (e_{trans} * \sin(E_0)))\} \quad (70)$$

Because this transfer starts at periapsis, so E0=0o. The transfer doesn't pass perigee, so K must equal zero

$$\therefore t_{trans} = \sqrt{\frac{a_{trans}^3}{\mu}} \{(E - (e_{trans} * \sin(E)))\} \quad (71)$$

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5. NUMERICAL RESULTS

The transfer is initiated by firing the space craft engine at low Earth orbit in order to accelerate it so that it will follow the elliptical orbit; this adds energy to the space craft's orbit. When the spacecraft has reached transfer orbit, its orbital speed (and hence its orbital energy) must be increased again in order to change the elliptic orbit to the final orbit. For simulation purposes five altitudes of low Earth orbits are considered as initial orbits for the Hohmann transfer, starting from altitude of 500 km up to 1500 km which are typical for LEO satellites to 36000 km altitude. These results are presented in table 1 and in figure 7 and in figure 8.

Table (1): comparison between Hohmann transfer, one tangent burn and Bi-elliptic transfer at different altitude.

Initial alt. (km)	Final alt. (km)	Hohmann transfer		One-tangent burn $v_{trans\ b}$=175 degree		Bi-elliptic Transfer Alt.=47836 km	
		ΔV (km/s)	T_{trans} (h)	ΔV(km/s)	T_{trans} (h)	ΔV(km/s)	T_{trans} (h)
500	36000	3.8195	5.34223	3.87076	4.76908	3.96491	22.04974
700	36000	3.74574	5.3748	3.79548	4.80757	3.89616	22.086
900	36000	3.67445	5.40744	3.72275	4.84589	3.82974	22.12233
1100	36000	3.60548	5.44014	3.65241	4.88404	3.76551	22.15871
1300	36000	3.53870	5.47291	3.58432	4.92205	3.70334	22.19515
1500	36000	3.47398	5.50574	3.51836	4.95992	3.6431	22.23166

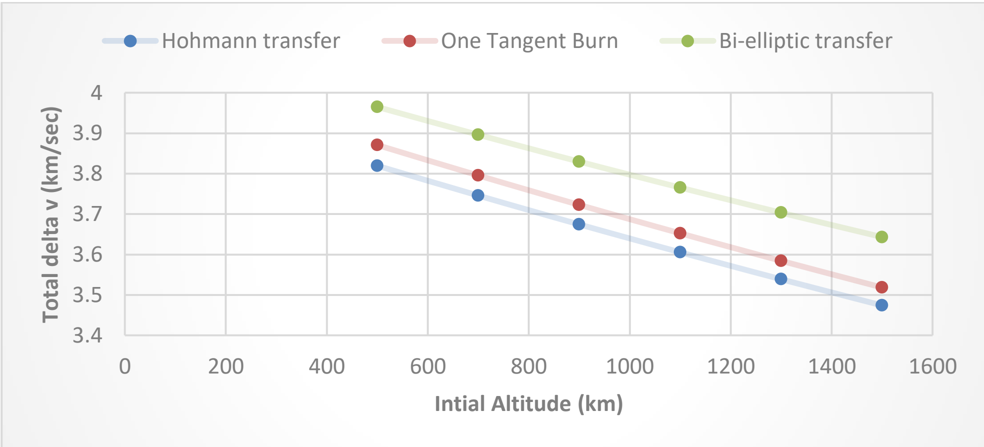

Figure (7): velocity variation under different initial orbit altitudes

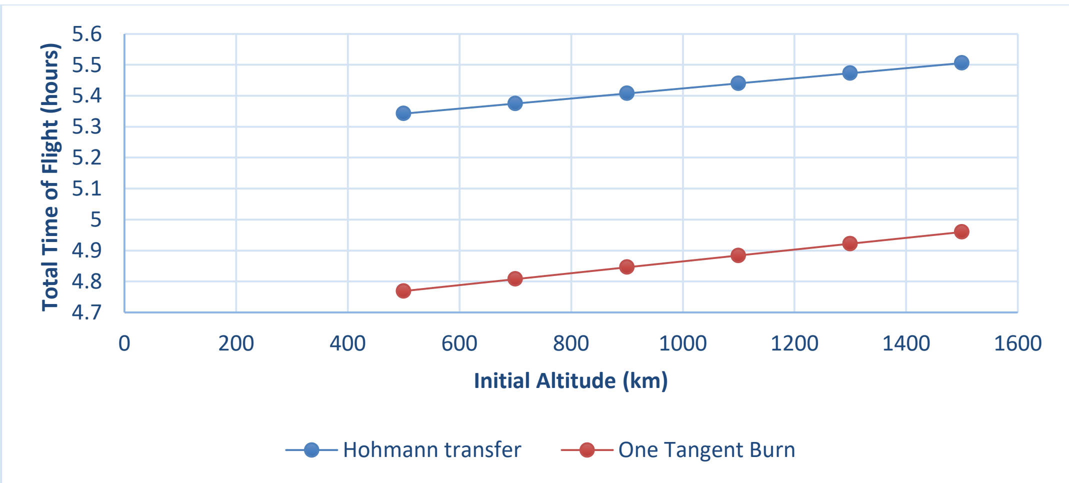

Figure (8): Time of flight variation under different initial orbit altitudes

Figure 7, confirms that as higher is the altitude of the low initial orbit, the lower velocity impulse is needed to the final destination orbit. Consequently less fuel is needed to be carried out on the satellite in case that the satellite initially is injected on the higher altitude toward the final orbit. The two impulse Hohmann transfer is more economical compare with three impulse (bi-elliptic transfer) and one tangent burn. Figure 8, confirms that one tangent burn is taking less time of flight compare with Hohmann transfer.

6. CONCLUSION

In orbital mechanics, the Hohmann transfer orbit is an elliptical orbit used to transfer between two elliptical orbits of different radii. If both orbits lie in the same plane, it is known as coplanar transfer. The orbital maneuver to perform the Hohmann transfer applies two engine impulses (thrusts), one to move a space craft onto the transfer orbit and a second to move off it.

One tangent Burn: as the name implies, a one-tangent burn has one tangential burn and one non-tangential burn. This method reduces the transfer time of the Hohmann techniques but increases ΔV requirements.

The two impulse Hohmann transfer is more economical compare with three impulse (bi-elliptic transfer) and one tangent burn. Consequently less fuel is needed to be carried out on the space craft in case that the spacecraft initially is injected on the higher altitude.

One tangent burn is the fastest method which take less time of flight compare with Hohmann transfer and Bi-elliptic transfer.

Through simulation results, it is also confirmed that for different altitudes of initial orbit, the first velocity impulse has faster decreasing gradient than the second velocity impulse.

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